Calculating Sensitivity Coefficients in Gradient-based History Matching

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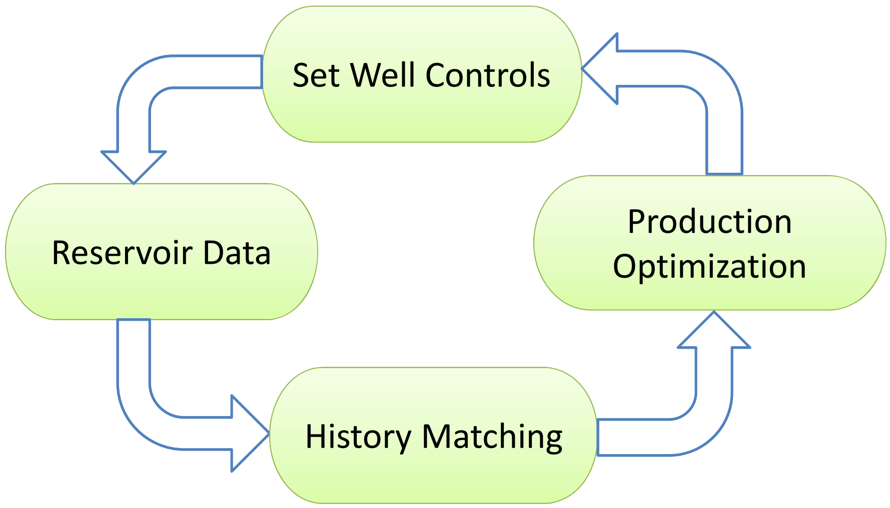
Nov 27, 2012

Outline

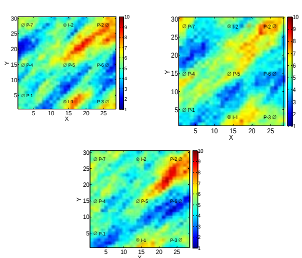
- Introduction & motivation
- Formulation of history matching problem
- Sensitivity matrix
- Examples
- Algorithms based on adjoint and gradient simulator methods

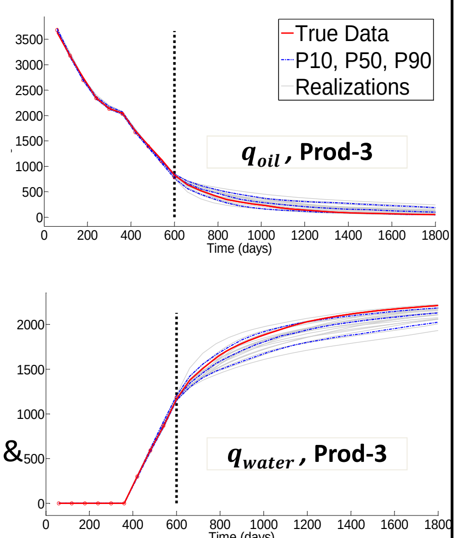
Closed-loop Reservoir Management



Motivation for History Matching

(Results from AD-GPRS)





- Generate a suite of plausible realizations
  - Future performance predictions
  - Models for optimal well control & optimal well placement

History Matching as an Inverse Problem

- In a **forward problem**
  - Physical properties are known
  - Response or outcome is calculated
- In an **inverse problem**
  - Physical properties of system are unknown (should be estimated)
  - Some noisy observed data (outcome) are given
  - Some prior knowledge about the model
- Inverse problems typically have nonunique solutions

History Matching in the Bayesian Framework

- Prior PDF $f(m) = a\exp\left(-O_m(m)\right),$

$$O_m(m) = \frac{1}{2}\left(m - m_{prior}\right)^T C_M^{-1}\left(m - m_{prior}\right)$$

m: N_m-dimensional vector of model parameter, $\ln k_h, \ln k_z, \ \phi$

C_M: Covariance matrix for prior pdf of m

History Matching in the Bayesian Framework

- Posterior PDF from Bayes theorem:

$$f(m|d_{obs}) = af(m)L(d_{obs}|m) = a\exp\big(-O(m)\big)$$

$$O(m) = \frac{1}{2}\left(m - m_{prior}\right)^T C_M^{-1}\left(m - m_{prior}\right)$$ ← Model mismatch term (prior)

$$+\frac{1}{2}(g(m) - d_{obs})^T C_D^{-1}(g(m) - d_{obs})$$ ← Data mismatch term (likelihood)

d_{obs} : N_d-dimensional vector of observed data: *BHP*, oil rate etc

$g(m)$: N_d-dimensional vector of predicted data: *BHP*, oil rate etc

C_D : (diagonal) covariance matrix for measurement errors

- Minimizing $O(m)$ gives the **m**aximum **a** **p**osteriori estimate (MAP)

Gradient-based History Matching

- Gradient-based methods:
  - Gauss-Newton (**GN**) & Levenberg-Marquardt (**LM**)
  - Nonlinear Conjugate Gradient (**PCG**).
  - Steepest Decent method.
  - quasi-Newton Methods, e.g. **LBFGS**
- **GN, LM:** $H\delta m = -\nabla O$
  - Forming the Hessian is computationally very expensive
  - Can use iterative Linear solvers, CG, MINRES
  - Can use **SVD parameterization**

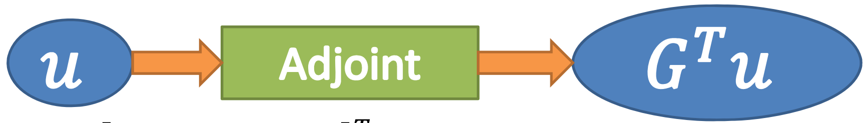
Gauss-Newton Hessian & Gradient

- Gauss-Newton $$H\,\delta m = -\nabla O$$
- Hessian: $$H_l = C_M^{-1} + G^T C_D^{-1} G$$
- Gradient: $$\nabla = -\{C_M^{-1}(m^l - m_{prior}) + G^T w\},$$ $$\nabla = -\{C_M^{-1}(m^l - m_{prior}) + G^T C_D^{-1}(g(m^l) - d_{obs})\},$$
- $\boldsymbol{G}$ is the $N_d \times N_m$ sensitivity matrix $$G = \left[\frac{\partial d_i}{\partial m_j}\right]$$

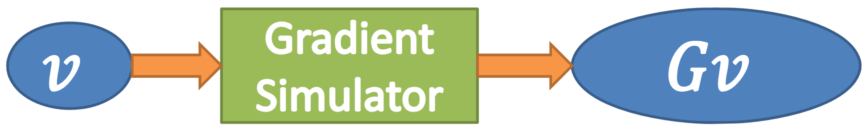
The Sensitivity matrix

- j-th column of G contains the sensitivity of all predicted data with respect to the j-th model parameter.
- i-th row of G contains the sensitivity of i-th predicted data with respect to all model parameters.
- $G \times v$: with the “gradient simulator method”
- $G^T \times u$: with the “adjoint method”.
- G can be computed with
  - N_d adjoint solutions
  - N_m applications of gradient simulator method.

The Sensitivity matrix

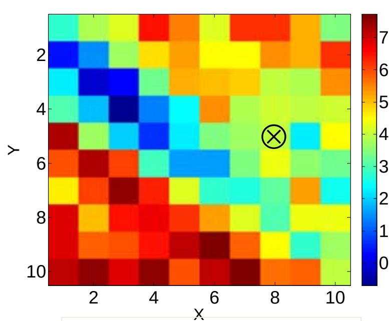


- If $u = [0, \dots, 0,1,0, \dots 0]^T$, then adjoint gives one row of G.
- If $u = C_D^{-1}(g(m) - d_{obs})$, then adjoint gives the gradient!



- If $u = [0, \dots, 0,1,0, \dots 0]^T$, then gradient simulator method gives one column of G.

2D Example (Single Injector)

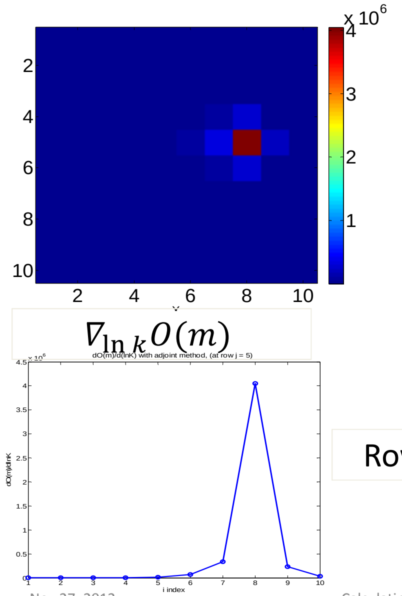


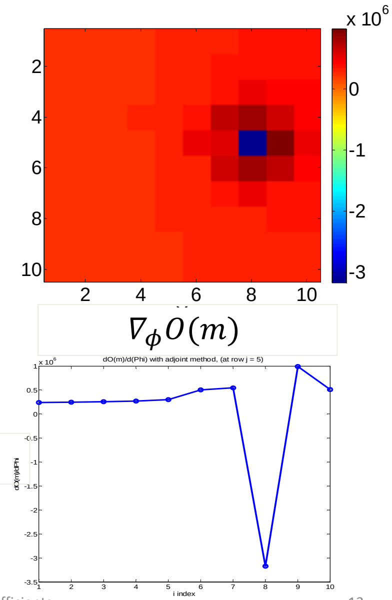
True $\ln(k)$

time	BHP Observed	*$g(m_{prior})$ Predicted*
30	8317	6875
60	8644	7163
90	8849	7422
120	9145	7696
150	9504	8019

- Initial Pressure: $P_i = 4800$ psi
- Observed Data: BHP **N_d =5**
- Model Parameters: $\ln(k)$-porosity , **N_m=200**
- $O(m) = \sum_{i=1}^{5} \frac{1}{\sigma_i^2} \left(d_{pred}^i - d_{obs}^i\right)^2$

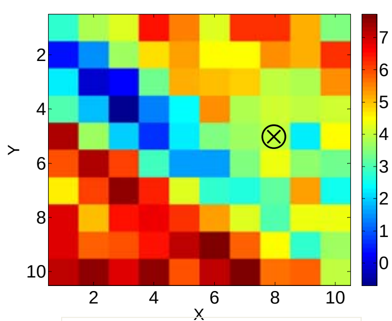
Gradient Computed by Adjoint at Uniform Estimate





Row $i = 5$

2D Example (Single Producer)

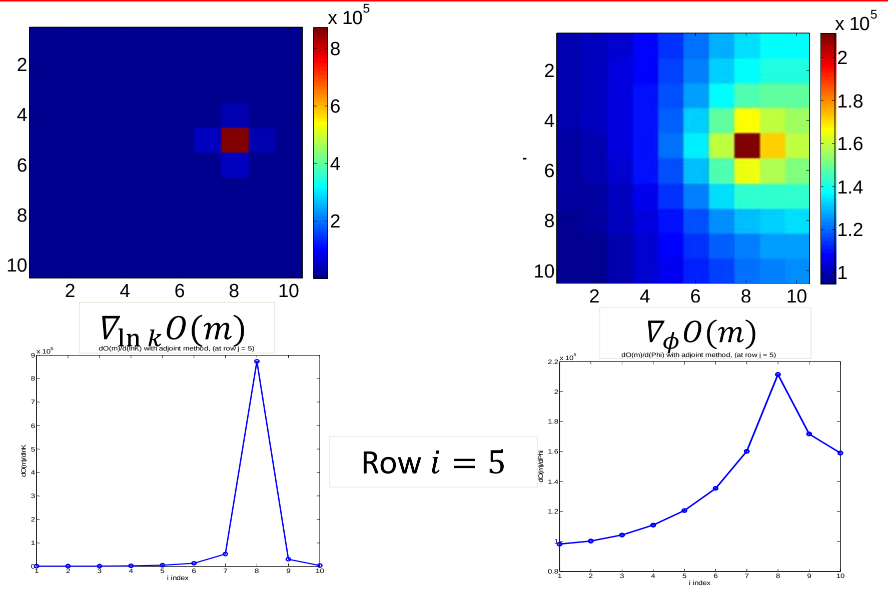


True $\ln(k)$

time	BHP Observed	*$g(m_{prior})$ Predicted*
30	3236	3826
60	2931	3565
90	2647	3304
120	2367	3037
150	2080	2765

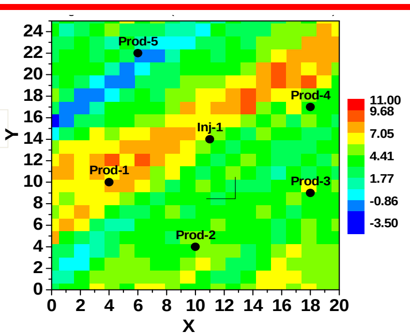
- Initial Pressure: $P_i = 4800$ psi
- Observed Data: **Oil rate** **N_d =5**
- Model Parameters: $\ln(k)$-porosity , **N_m=200**
- $O(m) = \sum_{i=1}^{5} \frac{1}{\sigma_i^2} \left(d_{pred}^i - d_{obs}^i\right)^2$

Gradient Computed by Adjoint at Uniform Estimate



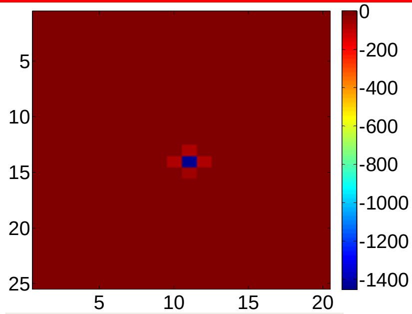
2D Example (20×25)

True $\ln(k)$

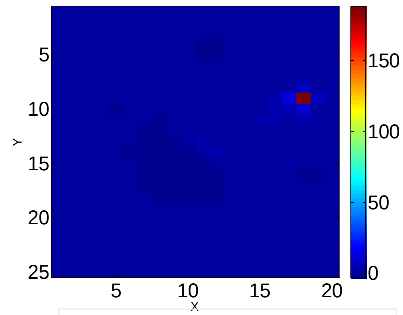


- Observed Data: BHP of Prod's and Inj, N_d = 726
- Model Parameters: $\ln(k)$, N_m = 500
- G : 726×500 $(N_d \times N_m)$ sensitivity matrix
- Compute sensitivity of BHP of Inj-1 and Prod-3 at t= 1980 Days

Sensitivity of One Specific Data By Adjoint



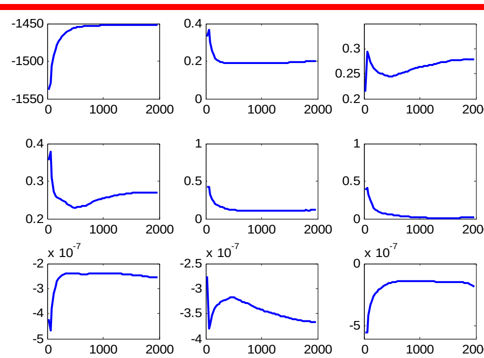
$\nabla_{\ln k}(p_{wf,\,t=1980,\,inj})$



$\nabla_{\ln k}(p_{wf,\,t=1980,\,prod3})$

- Computing $\nabla_{\ln k}(p_{wf,\,t=1980,\,inj}) = \frac{\partial(p_{wf,\,t=1980,\,inj})}{\partial \ln k}$ involves one adjoint solution. Same about the right figure.

Sensitivity of All Data w.r.t. One Specific Parameter



$$\frac{\partial g}{\partial \ln k\ (inj)}$$

- Sensitivity of all data w.r.t $\ln k$ of the injector gridblock, with direct method, from upper left to lower right, data represent p_{wf} of Inj-1, p_{wf} of prod-1, p_{wf} of prod-2, p_{wf} of prod-3, p_{wf} of prod-4, p_{wf} of prod-5, q_{water} of prod-2, q_{water} of prod-4, q_{water} of prod-5

Truncated SVD Parameterization

- Lanczos Algorithm: Iteratively approximates the largest singular values of G without explicit knowledge of the matrix.
- Requires Gv (gradient simulator method) and $G^T u$ (adjoint solution) for vectors v and u, (Rodrigues (2006), Tavakoli et al (2009,2010))
- Apply Lanczos algorithm to $G_D = C_D^{-\frac{1}{2}} G C_M^{\frac{1}{2}}$

$$G_D \approx U_p \Lambda_p V_p^T$$

Levenberg-Marquardt with Truncated SVD

- Modified Levenberg-Marquardt algorithm (to generate MAP estimate):

$$[(\gamma_l + 1) C_M^{-1} + G_l^T C_D^{-1} G_l] \delta m^{l+1} = -\{C_M^{-1}(m^l - m_{prior}) + G_l^T C_D^{-1}(g(m^l) - d_{obs})\},$$

- LM for the dimensionless model:

$$[(1+\gamma_l) I_{N_m} + G_D^T G_D] \delta \tilde{m}^{l+1} = -[\tilde{m}^l + G_D^T C_D^{-1/2}(g(m^l) - d_{obs})].$$

- Dimensionless domain:

$$\tilde{m} = C_M^{-\frac{1}{2}}(m^l - m_{prior}), \qquad \delta\tilde{m}^{l+1} = C_M^{-\frac{1}{2}} \delta m^{l+1}$$

Levenberg-Marquardt with Truncated SVD

- Hessian:

$$G_D^T G_D \approx V_p \Lambda_p^2 V_p^T$$

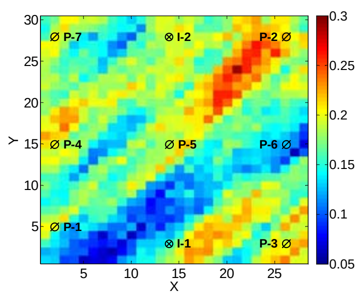
- Eigenvectors of Hessian = Right singular vectors of G_D
- Eigenvalues of Hessian = squares of the singular values of G_D

- Parameterize the change in the model, δm, in terms of the eigenvectors of Hessian associated with the largest eigenvalues (α_k can be easily computed):

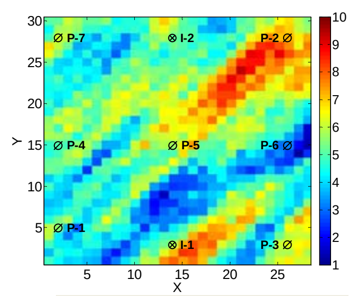
$$\delta m = \sum_{1}^{p} \alpha_k v_k$$

2D Example (28 × 30)

- Model Parameters: $\ln(k)$
- History matching 1800 days of rate data
- Producers Total Liquid Rate: 300 - 200 STB/Day
- Observed Data: BHP's, qw(1), qw(3),qw(7), N_d =720
- Model Parameters: ln(k)-porosity , N_m=1680

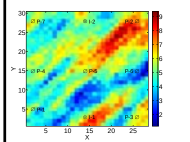


True Porosity

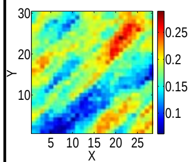


True $\ln(k)$

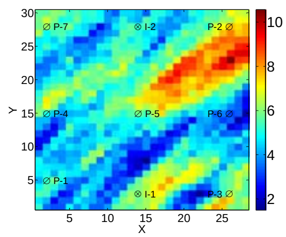
An Updated Realization from Levenberg-Marqurdt with Truncated SVD SVD-RML



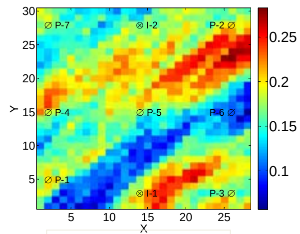
True Ln(k)



True Porosity



Ln(k)



Porosity

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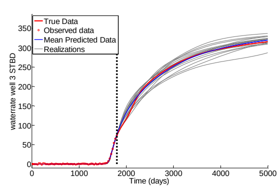
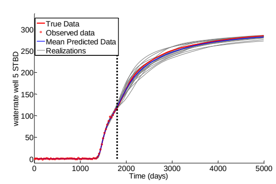
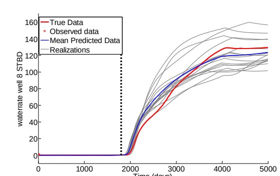
Performance predictions with 16 realizations of the posterior



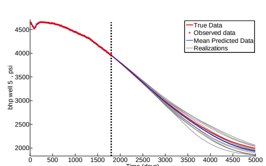
q_w Prod-1



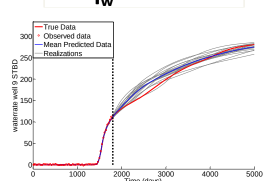
q_w Prod-3



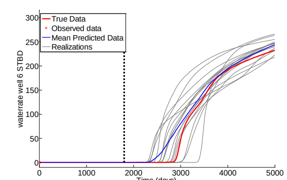
q_w Prod-6



p_{wf} Prod-4



q_w Prod-7



q_w Prod-4

- Red: Truth; blue: mean of Predictions.

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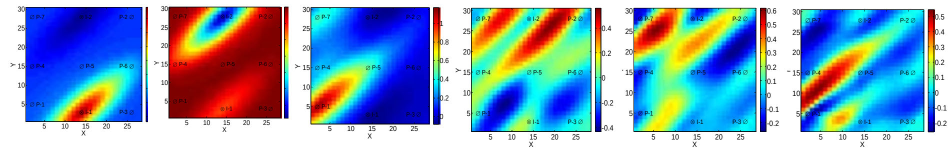
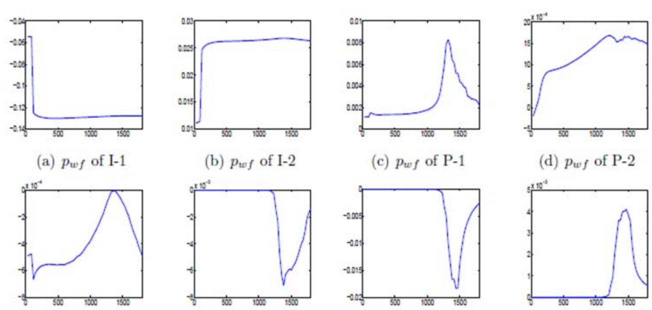
Eigenvectors of the Hessian (left multiplied by $C_M^{1/2}$)



$Ln(k), Lv_1$ | $Ln(k), Lv_2$ | $Ln(k), Lv_3$ | $Ln(k), Lv_{16}$ | $Ln(k), Lv_{17}$ | $Ln(k), Lv_{20}$

$\lambda_1 = 7474$ $\lambda_2 = 7108$ $\lambda_3 = 2621$ $\lambda_{16} = 140$ $\lambda_{17} = 117$ $\lambda_{20} = 72$

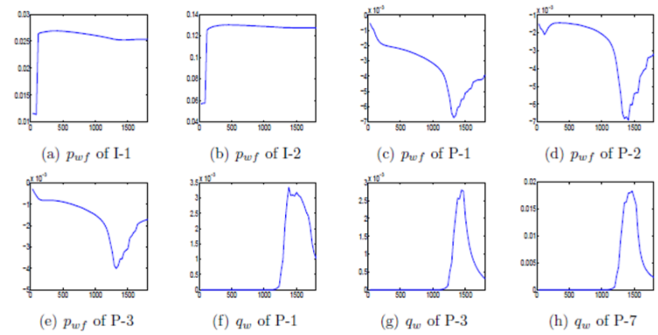
1st Left Singular Vector at the First Iteration for the MAP Estimate



(a) p_{wf} of I-1 (b) p_{wf} of I-2 (c) p_{wf} of P-1 (d) p_{wf} of P-2

(e) p_{wf} of P-4 (f) q_w of P-1 (g) q_w of P-3 (h) q_w of P-7

2^{nd} Left Singular Vector at the First Iteration for the MAP Estimate



Thank you!